A general class of surrogate functions for stable and efficient reinforcement learning

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Theory of RL Workshop, Alberta

Motivation

- Policy gradient (PG) methods based on REINFORCE:
  - Each policy update requires recomputing the policy gradient.
  - ✓ Theoretical guarantees [Agarwal et al., 2020] with function approximation.
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No systematic way to design theoretically principled surrogate functions, or a unified framework to analyze their properties.

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- Formal problem definition
- Functional mirror ascent for policy gradient (FMA-PG) framework
- Theoretical guarantees
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- Generalizing the FMA-PG framework
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- Expected discounted return for π: $J(\pi) = \mathbb{E}_{\mathbf{s_0}, \mathbf{a_0}, \ldots}[\sum_{\tau=0}^{\infty} \gamma^\tau r(s_\tau, a_\tau)]$, where $s_0 \sim \rho, a_\tau \sim p^\pi(\cdot|s_\tau)$, and $s_{\tau+1} \sim p(\cdot|s_\tau, a_\tau)$.
- **Objective**: Given a set of feasible policies Π, $\max_{\pi \in \Pi} J(\pi)$. $\pi^* := \arg\max_{\pi \in \Pi} J(\pi)$.

Functional representation vs Policy parameterization

- **Functional representation**: Specifies a policy's sufficient statistics and is implicit.

  Examples:

  - *Direct functional representation*: Conditional distribution over actions $p^\pi(\cdot|s)$ for each s.
  - *Softmax functional representation*: Logits $z^\pi(s, a)$ such that $p^\pi(a|s) = \frac{\exp(z^\pi(s,a))}{\sum_{a'} \exp(z^\pi(s,a'))}$.

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- **Policy parameterization**: Practical realization of the sufficient statistics. Determines Π (the set of feasible policies). *Examples*:
  - *Tabular parameterization* for the direct functional representation: $p^{\pi}(a|s) = \theta(s, a)$.
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- **Standard PG approach**: Use a model (with parameters θ) to parameterize (the functional representation of) π and directly optimize $J(\pi(\theta))$ w.r.t. θ.

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- For a strictly convex, differentiable function Φ (*mirror map*), $D_\Phi(\pi, \pi')$ is the *Bregman divergence* between policies π and π'. $D_\Phi(\pi, \pi') := \Phi(\pi) - \Phi(\pi') - \langle \nabla\Phi(\pi'), \pi - \pi' \rangle$.

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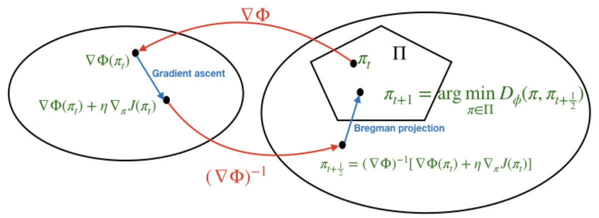
In each iteration $t \in [T]$ of *functional mirror ascent* (FMA), with *step-size* η,

$$\pi_{t+1/2} = (\nabla\Phi)^{-1} \left(\nabla\Phi(\pi_t) + \eta \nabla_\pi J(\pi_t)\right) \quad ; \quad \pi_{t+1} = \arg\min_{\pi \in \Pi} D_\Phi(\pi, \pi_{t+1/2})$$



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FMA-PG framework

- The complexity of the projection onto Π depends on the parameterization. *Examples*:
  - For a tabular parameterization, Π allows all memoryless policies.
  - For a linear parameterization, Π is restricted, but is a convex set in θ.
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If Π consists of policies realizable by a parametric model, then

$$\pi_{t+1} = \arg\min_{\pi \in \Pi} D_\Phi(\pi, \pi_{t+1/2}) = \arg\min_{\theta \in \mathbb{R}^d} D_\Phi(\pi(\theta), \pi_{t+1/2}) \qquad \text{(\textbf{Reparameterization})}$$

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With this reparameterization, the FMA update can be rewritten as:

$$\pi_{t+1} = \pi(\theta_{t+1}) \quad ; \quad \theta_{t+1} = \arg\max_{\theta \in \mathbb{R}^d} \underbrace{\left[J(\pi(\theta_t)) + \langle \pi(\theta) - \pi(\theta_t), \nabla_\pi J(\pi(\theta_t)) \rangle - \frac{1}{\eta} D_\Phi(\pi(\theta), \pi(\theta_t)) \right]}_{\text{Surrogate function } \ell_t^{\pi,\Phi,\eta}(\theta)}$$

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$\ell_t(\theta)$ is non-concave in general, and we optimize it using a gradient-based method.

FMA-PG framework - Putting everything together

Algorithm 1: Generic policy optimization

Input: π (functional representation), θ_0 (initial policy parameterization), T (PG iterations), m (inner-loops), η (step-size for functional update), α (step-size for parametric update)

for $t \leftarrow 0$ **to** $T-1$ **do**
- Compute $\nabla_\pi J(\pi_t)$ and form the surrogate $\ell_t^{\pi,\Phi,\eta}(\theta)$.
- Initialize inner-loop: $\omega_0 = \theta_t$
- **for** $k \leftarrow 0$ **to** m **do**
  - $\omega_{k+1} = \omega_k + \alpha \nabla_\omega \ell_t^{\pi,\Phi,\eta}(\omega_k)$ /* `Off-policy actor updates` */
- $\theta_{t+1} = \omega_m$
- $\pi_{t+1} = \pi(\theta_{t+1})$

Return θ_T

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- Recall that, $\ell_t(\theta) = J(\pi(\theta_t)) + \langle \pi(\theta) - \pi(\theta_t), \nabla_\pi J(\pi(\theta_t)) \rangle - \frac{1}{\eta} D_\Phi(\pi(\theta), \pi(\theta_t))$.
- Sufficient conditions to ensure monotonic policy improvement, i.e. $J(\pi_{t+1}) \geq J(\pi_t)$:
  (i) $\ell_t(\theta_{t+1}) \geq \ell_t(\theta_t)$, [Inner-loop improves the surrogate value]
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- (ii) is satisfied by setting the *functional* step-size η according to the relative smoothness of $J(\pi)$ w.r.t D_Φ. Specifically, any η that ensures $J + \frac{1}{\eta}\Phi$ is a convex function guarantees (ii).

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Setting η for the direct functional representation with negative entropy mirror map

For any policy parameterization, $\forall\theta$, $J(\pi(\theta)) \geq \ell_t^{\pi,\mathrm{NE},\eta}(\theta)$ for $\eta \leq \frac{(1-\gamma)^3}{2\gamma|A|}$.

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  - similar to uniform TRPO [Shani et al., 2020] and Mirror Descent Modified Policy Iteration [Geist et al., 2019].

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× Surrogate involves the importance-sampling ratio $\frac{p^\pi(a|s,\theta)}{p^\pi(a|s,\theta_t)}$ that could be potentially large.

× Surrogate involves the reverse KL divergence making it *mode seeking* hindering exploration [Mei et al., 2019].

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For the log-sum-exp mirror-map i.e. when $\phi_z(z(s, \cdot)) = \log\left(\sum_a \exp(z^{\pi}(s, a))\right)$,

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Setting η for the softmax functional representation with log-sum-exp mirror map

For any policy parameterization, $\forall\theta$, $J(\pi(\theta)) \geq \ell_t^{\pi,\mathrm{LSE},\eta}(\theta)$ for $\eta \leq 1-\gamma$.

Instantiating FMA-PG - Softmax functional representation

The surrogate can be rewritten as

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Compared to TRPO: $\max_{\theta\in\mathbb{R}^d}\mathbb{E}_{(s,a)\sim\mu^{\pi_t}}[A^{\pi_t}(s,a)\,\frac{p^{\pi}(a|s,\theta)}{p^{\pi}(a|s,\theta_t)}]$ s.t. $\mathbb{E}_{s\sim d^{\pi_t}}[\text{KL}(p^{\pi_t}(\cdot|s,\theta_t)||p^{\pi}(\cdot|s,\theta))] \leq \delta$

- $\ell_t^{\pi,\text{LSE},\eta}(\theta)$ involves the log of the importance sampling ratio, and enforces proximity between policies using a regularization (with parameter $1/\eta$) rather than a constraint.
- we set the step-sizes that ensure monotonic policy improvement for any policy parameterization and any number of inner-loops.

Conclusion

- ✓ Used functional mirror ascent to propose FMA-PG, a systematic way to define surrogate functions for generic policy optimization. Ensures monotonic policy improvement for arbitrary policy parameterization.
- ✓ Can use the FMA-PG framework to "lift" existing theoretical guarantees [Mei et al., 2020, Xiao, 2022] for policy optimization algorithms in the tabular setting to use off-policy updates and function approximation[1].
- ✓ Show experimental evidence that on simple tabular MDPs, the algorithms instantiated with FMA-PG are competitive with popular PG algorithms such as TRPO, PPO. The framework suggests sPPO that out-performs PPO on the MuJoCo suite.

[1] Under some assumptions

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Problem: FMA-PG relies on estimates of the true gradient $\nabla_\pi J(\pi)$, which invovles either the action-value Q^π or the advantage A^π functions. Typically, these functions can only be estimated, making FMA-PG impractical in realistic settings.

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Proposition: For any gradient estimator $\hat{g}_t$ at iteration t, for any $c > 0$ and η such that $J + \frac{1}{\eta}\Phi$ is convex in π, if Φ^* is the Fenchel-conjugate of Φ, we have **Inequality I**: $J(\pi) - J(\pi_t) \geq$

$$\underbrace{\langle \hat{g}_t, \pi(\theta) - \pi_t \rangle - \left(\frac{1}{\eta} + \frac{1}{c}\right) D_\Phi(\pi(\theta), \pi_t) -}_{\text{Surrogate function that can be maximized as before}} \underbrace{\frac{1}{c} D_{\Phi^*}\left(\nabla \Phi(\pi_t) - c[\nabla J(\pi_t) - \hat{g}_t], \nabla \Phi(\pi_t)\right)}_{\text{Error in } Q^\pi \text{ or } A^\pi \text{ estimation. Can be minimized by training a critic}}.$$

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Directly gives a joint-objective for an actor-critic algorithm where both components are learned to minimize the same lower-bound.

Algorithm 2: Generic actor-critic algorithm

Input: π (choice of functional representation), θ_0 (initial policy parameters), $\omega_{(-1)}$ (initial critic parameters), T (AC iterations), m_a (actor inner-loops), m_c (critic inner-loops), η (functional step-size for actor), c (trade-off parameter), α_a (parametric step-size for actor), α_c (parametric step-size for critic)

Initialization: $\pi_0 = \pi(\theta_0)$

for $t \leftarrow 0$ **to** $T-1$ **do**

Estimate $\widehat{\nabla_\pi} J(\pi_t)$ and form $\mathcal{L}_t(\omega) := \frac{1}{c} D_{\Phi^*}\left(\nabla\Phi(\pi_t) - c\,[\widehat{\nabla_\pi} J(\pi_t) - \hat{g}_t(\omega)], \nabla\Phi(\pi_t)\right)$

Initialize inner-loop: $\upsilon_0 = \omega_{t-1}$

for $k \leftarrow 0$ **to** $m_c - 1$ **do**

$\upsilon_{k+1} = \upsilon_k - \alpha_c \nabla_\upsilon \mathcal{L}_t(\upsilon_k)$ /* Critic Updates */

$\omega_t = \upsilon_{m_c}$; $\hat{g}_t = \hat{g}_t(\omega_t)$

Form $\ell_t(\theta) := \langle \hat{g}_t, \pi(\theta) - \pi_t \rangle - \left(\frac{1}{\eta} + \frac{1}{c}\right) D_\Phi(\pi(\theta), \pi_t)$

Initialize inner-loop: $\nu_0 = \theta_t$

for $k \leftarrow 0$ **to** $m_a - 1$ **do**

$\nu_{k+1} = \nu_k + \alpha_a \nabla_\nu \ell_t(\nu_k)$ /* Off-policy actor updates */

$\theta_{t+1} = \nu_{m_a}$; $\pi_{t+1} = \pi(\theta_{t+1})$

Return $\pi_T = \pi(\theta_T)$

Generalizing the FMA-PG framework – Guarantees

Proposition: For any policy representation and any policy or critic parameterization, there exists a (θ, c) pair that makes the RHS of **inequality (I)** strictly positive, and hence guarantees monotonic policy improvement $(J(\pi_{t+1}) > J(\pi_t))$, if and only if the critic error satisfies a certain technical condition that depends on the policy parameterization and the mirror map.

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Special case: For the tabular policy parameterization with the Euclidean mirror map, this condition is equivalent to: $\|\hat{g}_t\|_2^2 > \|\nabla J(\pi_t) - \hat{g}_t\|_2^2$.

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Proposition: For any critic error, policy representation and mirror map Φ such that (i) $J + \frac{1}{\eta}\Phi$ is convex in π, any policy parameterization such that (ii) $\ell_t(\theta)$ is smooth w.r.t θ and satisfies the Polyak-Lojasiewicz (PL) condition, for $c > 0$, we show that Algorithm 1 converges to a neighbourhood of the stationary point at an $O(1/T)$ rate. The neighbourhood depends on the critic error and the number of off-policy actor updates.

Generalizing the FMA-PG framework – Instantiation

Proposition: For the direct representation and negative entropy mirror map, $c > 0$, $\eta \leq \frac{(1-\gamma)^3}{2\gamma\,|A|}$,

$$J(\pi) - J(\pi_t) \geq C + \mathbb{E}_{s \sim d^{\pi_t}} \left[\mathbb{E}_{a \sim p^{\pi_t}(\cdot|s)} \left[\frac{p^{\pi}(a|s)}{p^{\pi_t}(a|s)} \left(\hat{Q}^{\pi_t}(s,a) - \left(\frac{1}{\eta} + \frac{1}{c} \right) \log \left(\frac{p^{\pi}(a|s)}{p^{\pi_t}(a|s)} \right) \right) \right] \right]$$

$$- \mathbb{E}_{s \sim d^{\pi_t}} \left[\mathbb{E}_{a \sim p^{\pi_t}(\cdot|s)} [Q^{\pi_t}(s,a) - \hat{Q}^{\pi_t}(s,a)] + \frac{1}{c} \log \left(\mathbb{E}_{a \sim p^{\pi_t}(\cdot|s)} \left[\exp \left(-c\,[Q^{\pi_t}(s,a) - \hat{Q}^{\pi_t}(s,a)] \right) \right] \right) \right]$$

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- We refer to this as the decision-aware critic loss since minimizing it directly improves the lower-bound on $J(\pi)$ and can result in improving the policy. This is especially important when using a critic model with limited capacity.

Generalizing the FMA-PG framework – Instantiation

Consider a two-armed bandit example with deterministic rewards where arm 1 is optimal and has a reward $r_1 = Q_1 = 2$ whereas arm 2 has reward $r_2 = Q_2 = 1$. Using a linear parameterization for the critic, Q function is estimated as: $\hat{Q} = x\,\omega$ where ω is the parameter to be learned and x is the feature of the corresponding arm. Let $x_1 = -2$ and $x_2 = 1$ implying that $\hat{Q}_1(\omega) = -2\omega$ and $\hat{Q}_2(\omega) = \omega$. Let p_t be the probability of pulling the optimal arm at iteration t, and consider minimizing two alternative objectives to estimate ω:

(1) Squared loss: $\omega_t^{(1)} := \arg\min \mathrm{TD}(\omega) := \arg\min \left\{ \frac{p_t}{2}\,[\hat{Q}_1(\omega) - Q_1]^2 + \frac{1-p_t}{2}\,[\hat{Q}_2(\omega) - Q_2]^2 \right\}$.

(2) Decision-aware critic loss: $\omega_t^{(2)} := \arg\min \mathcal{L}_t(\omega) := p_t\,[Q_1 - \hat{Q}_1(\omega)] + (1-p_t)\,[Q_2 - \hat{Q}_2(\omega)] + \frac{1}{c}\,\log\left(p_t\,\exp\left(-c\,[Q_1 - \hat{Q}_1(\omega)] + (1-p_t)\,\exp\left(-c\,[Q_2 - \hat{Q}_2(\omega)]\right)\right)\right]$.

Using the tabular parameterization for the actor, the policy update at iteration t is given by:

$p_{t+1} = \frac{p_t\,\exp(\eta \hat{Q}_1)}{p_t\,\exp(\eta\hat{Q}_1) + (1-p_t)\,\exp(\eta\hat{Q}_2)}$, where η is the functional step-size for the actor.

For $p_0 < \frac{2}{5}$, minimizing the squared loss results in convergence to the sub-optimal action, while minimizing the decision-aware loss (for any $c, p_0 > 0$) results in convergence to the optimal action.

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Consider a two-armed bandit example with deterministic rewards where arm 1 is optimal and has a reward $r_1 = Q_1 = 2$ whereas arm 2 has reward $r_2 = Q_2 = 1$. Using a linear parameterization for the critic, Q function is estimated as: $\hat{Q} = x\,\omega$ where ω is the parameter to be learned and x is the feature of the corresponding arm. Let $x_1 = -2$ and $x_2 = 1$ implying that $\hat{Q}_1(\omega) = -2\omega$ and $\hat{Q}_2(\omega) = \omega$. Let p_t be the probability of pulling the optimal arm at iteration t, and consider minimizing two alternative objectives to estimate ω:

(1) Squared loss: $\omega_t^{(1)} := \arg\min \text{TD}(\omega) := \arg\min \left\{ \frac{p_t}{2} [\hat{Q}_1(\omega) - Q_1]^2 + \frac{1-p_t}{2} [\hat{Q}_2(\omega) - Q_2]^2 \right\}$.

(2) Decision-aware critic loss: $\omega_t^{(2)} := \arg\min \mathcal{L}_t(\omega) := p_t\,[Q_1 - \hat{Q}_1(\omega)] + (1-p_t)\,[Q_2 - \hat{Q}_2(\omega)] + \frac{1}{c} \log\left(p_t \exp\left(-c\,[Q_1 - \hat{Q}_1(\omega)]\right) + (1-p_t) \exp\left(-c\,[Q_2 - \hat{Q}_2(\omega)]\right)\right)\Big]$.

Using the tabular parameterization for the actor, the policy update at iteration t is given by: $p_{t+1} = \frac{p_t \exp(\eta \hat{Q}_1)}{p_t \exp(\eta \hat{Q}_1) + (1-p_t) \exp(\eta \hat{Q}_2)}$, where η is the functional step-size for the actor.

For $p_0 < \frac{2}{5}$, minimizing the squared loss results in convergence to the sub-optimal action, while minimizing the decision-aware loss (for any $c, p_0 > 0$) results in convergence to the optimal action.

- Show similar results for the softmax functional representation.

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- Formal problem definition
- Functional mirror ascent for policy gradient (FMA-PG) framework
- Theoretical guarantees
- Instantiating the FMA-PG framework
- Generalizing the FMA-PG framework
- Conclusions and Future Work

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- ✓ Generalized FMA-PG to design a generic decision-aware actor-critic framework where the actor and critic are trained cooperatively to optimize a joint objective.
- ✓ Simple tabular experiments with a linear parameterization for the actor/critic demonstrate that being decision-aware is important when the critic is not as expressive.

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- ✓ Simple tabular experiments with a linear parameterization for the actor/critic demonstrate that being decision-aware is important when the critic is not as expressive.

- Prove rates of convergence to the optimal policy for the proposed AC algorithm.
- Benchmark the AC framework for complex deep RL environments.

Questions?

Papers: https://arxiv.org/abs/2108.05828, https://arxiv.org/abs/2305.15249

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Alekh Agarwal, Sham M. Kakade, Jason D. Lee, and Gaurav Mahajan. Optimality and approximation with policy gradient methods in Markov decision processes. In *Conference on Learning Theory (COLT)*, pages 64–66, 2020.

Matthieu Geist, Bruno Scherrer, and Olivier Pietquin. A theory of regularized Markov decision processes. In *International Conference on Machine Learning*, pages 2160–2169. PMLR, 2019.

Chloe Ching-Yun Hsu, Celestine Mendler-Dünner, and Moritz Hardt. Revisiting design choices in proximal policy optimization. *arXiv preprint arXiv:2009.10897*, 2020.

Sham Kakade. A natural policy gradient. In *NIPS*, volume 14, pages 1531–1538, 2001.

Jincheng Mei, Chenjun Xiao, Ruitong Huang, Dale Schuurmans, and Martin Müller. On principled entropy exploration in policy optimization. In *IJCAI*, pages 3130–3136, 2019.

Jincheng Mei, Chenjun Xiao, Csaba Szepesvari, and Dale Schuurmans. On the global convergence rates of softmax policy gradient methods. In *International Conference on Machine Learning*, pages 6820–6829. PMLR, 2020.

Lior Shani, Yonathan Efroni, and Shie Mannor. Adaptive trust region policy optimization: Global convergence and faster rates for regularized mdps. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 5668–5675, 2020.

Manan Tomar, Lior Shani, Yonathan Efroni, and Mohammad Ghavamzadeh. Mirror descent policy optimization. *arXiv preprint arXiv:2005.09814*, 2020.

Ronald J Williams and Jing Peng. Function optimization using connectionist reinforcement learning algorithms. *Connection Science*, 3(3):241–268, 1991.

Lin Xiao. On the convergence rates of policy gradient methods. *Journal of Machine Learning Research*, 23(282): 1–36, 2022.